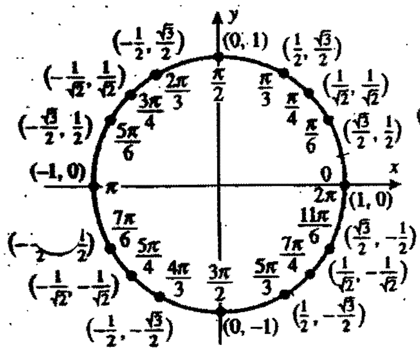
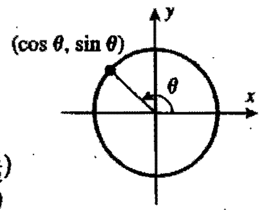
Math 253 Theorems



TRIGONOMETRY REVIEW



PYTHAGOREAN IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

SIGN IDENTITIES

$\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad \sec(-\theta) = \sec\theta$ $\quad \cot(-\theta) = -\cot\theta$

COMPLEMENT IDENTITIES

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

SUPPLEMENT IDENTITIES

$\sin(\pi - \theta) = \sin\theta$ $\quad \cos(\pi - \theta) = -\cos\theta$ $\quad \tan(\pi - \theta) = -\tan\theta$

$\csc(\pi - \theta) = \csc\theta$ $\quad \sec(\pi - \theta) = -\sec\theta$ $\quad \cot(\pi - \theta) = -\cot\theta$

$\sin(\pi + \theta) = -\sin\theta$ $\quad \cos(\pi + \theta) = -\cos\theta$ $\quad \tan(\pi + \theta) = \tan\theta$

$\csc(\pi + \theta) = -\csc\theta$ $\quad \sec(\pi + \theta) = -\sec\theta$ $\quad \cot(\pi + \theta) = \cot\theta$

ADDITION FORMULAS

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

DOUBLE-ANGLE FORMULAS

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\quad \cos 2\alpha = 2\cos^2\alpha - 1$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\quad \cos 2\alpha = 1 - 2\sin^2\alpha$

HALF-ANGLE FORMULAS

$$\sin^2\frac{\alpha}{2} = \frac{1 - \cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1 + \cos\alpha}{2}$$

EXPRESSION IN THE INTEGRAND	SUBSTITUTION	RESTRICTION ON θ
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\begin{cases} 0 \le \theta < \pi/2 & (\text{if } x \ge a) \\ \pi/2 < \theta \le \pi & (\text{if } x \le -a) \end{cases}$