

# TABLE OF INTEGRALS

1.  $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx.$
2.  $\int cf(x) dx = c \int f(x) dx.$
3.  $\int f(g(x))g'(x) dx = \int f(u) du \Big|_{u=g(x)}$
4.  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, (n \neq -1).$
5.  $\int \frac{1}{x} dx = \ln|x| + C.$
6.  $\int \sin x dx = -\cos x + C.$
7.  $\int \cos x dx = \sin x + C.$
8.  $\int \sec^2 x dx = \tan x + C.$
9.  $\int \csc^2 x dx = -\cot x + C.$
10.  $\int \sec x \tan x dx = \sec x + C.$
11.  $\int \csc x \cot x dx = -\csc x + C.$
12.  $\int e^x dx = e^x + C.$
13.  $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C.$
14.  $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C.$
15.  $\int \frac{1}{x \sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + C.$
16.  $\int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx.$
17.  $\int \frac{1}{x \sqrt{ax+b}} dx = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C, (b > 0).$
18.  $\int \frac{1}{x \sqrt{ax+b}} dx = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax+b}{-b}} + C, (b < 0).$
19.  $\int \frac{1}{x^n \sqrt{ax+b}} dx = -\frac{1}{b(n-1)} \frac{\sqrt{ax+b}}{x^{n-1}} - \frac{(2n-3)a}{(2n-2)b} \int \frac{1}{x^{n-1} \sqrt{ax+b}} dx, (n \neq 1).$
20.  $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{1}{x \sqrt{ax+b}} dx.$
21.  $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C.$
22.  $\int \frac{1}{(ax+b)(cx+d)} dx = \frac{1}{bc-ad} \ln \left| \frac{cx+d}{ax+b} \right| + C, (bc-ad \neq 0).$
23.  $\int \frac{x}{(ax+b)(cx+d)} dx = \frac{1}{bc-ad} \left\{ \frac{b}{a} \ln |ax+b| - \frac{d}{c} \ln |cx+d| \right\} + C, (bc-ad \neq 0).$
24.  $\int \frac{1}{(ax+b)^2(cx+d)} dx = \frac{1}{bc-ad} \left\{ \frac{1}{ax+b} + \frac{c}{bc-ad} \ln \left| \frac{cx+d}{ax+b} \right| \right\} + C, (bc-ad \neq 0).$
25.  $\int \frac{x}{(ax+b)^2(cx+d)} dx = -\frac{1}{bc-ad} \left\{ \frac{b}{a(ax+b)} + \frac{d}{bc-ad} \ln \left| \frac{cx+d}{ax+b} \right| \right\} + C, (bc-ad \neq 0).$

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26.  $\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}| + C.$
27.  $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}| + C.$
28.  $\int x^2 \sqrt{x^2 \pm a^2} dx = \frac{x}{8} (2x^2 \pm a^2) \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 \pm a^2}| + C.$
29.  $\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}| + C.$
30.  $\int (x^2 \pm a^2)^{3/2} dx = x(x^2 \pm a^2)^{3/2} - 3 \int x^2 \sqrt{x^2 \pm a^2} dx.$
31.  $\int \frac{1}{(x^2 \pm a^2)^{3/2}} dx = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}} + C.$
32.  $\int \frac{x^2}{(x^2 \pm a^2)^{3/2}} dx = \frac{-x}{\sqrt{x^2 \pm a^2}} + \ln |x + \sqrt{x^2 \pm a^2}| + C.$
33.  $\int \frac{1}{x^2 \sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C.$
34.  $\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = - \frac{\sqrt{x^2 \pm a^2}}{x} + \ln |x + \sqrt{x^2 \pm a^2}| + C.$
35.  $\int \frac{\sqrt{x^2 \pm a^2}}{x} dx = \sqrt{x^2 \pm a^2} \pm a^2 \int \frac{1}{x \sqrt{x^2 \pm a^2}} dx.$
36.  $\int \frac{1}{x \sqrt{x^2 + a^2}} dx = - \frac{1}{a} \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right| + C.$
37.  $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C.$
38.  $\int x^2 \sqrt{a^2 - x^2} dx = - \frac{x}{4} (a^2 - x^2)^{3/2} + \frac{a^2}{4} \int \sqrt{a^2 - x^2} dx.$
39.  $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = - \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C.$
40.  $\int (a^2 - x^2)^{3/2} dx = \frac{x}{4} (a^2 - x^2)^{3/2} + \frac{3a^2}{4} \int \sqrt{a^2 - x^2} dx.$

41.  $\int \frac{1}{(a^2 - x^2)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C.$
42.  $\int \frac{x^2}{(a^2 - x^2)^{3/2}} dx = \frac{x}{\sqrt{a^2 - x^2}} - \sin^{-1} \frac{x}{a} + C.$
43.  $\int \frac{1}{x \sqrt{a^2 - x^2}} dx = - \frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C.$
44.  $\int \frac{1}{x^2 \sqrt{a^2 - x^2}} dx = - \frac{\sqrt{a^2 - x^2}}{a^2 x} + C.$
45.  $\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C.$
46.  $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = - \frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \frac{x}{a} + C.$
47.  $\int \frac{1}{(x^2 + a^2)^n} dx$   
 $= \frac{1}{2(n-1)a^n} \left\{ \frac{x}{(x^2 + a^2)^{n-1}} + (2n-3) \int \frac{1}{(x^2 + a^2)^{n-1}} dx \right\}, (n \neq 1).$
48.  $\int x \sin x dx = \sin x - x \cos x + C.$
49.  $\int x^n \sin x dx = -x^n \cos x + nx^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x dx.$
50.  $\int x \cos x dx = \cos x + x \sin x + C.$
51.  $\int x^n \cos x dx = x^n \sin x + nx^{n-1} \cos x - n(n-1) \int x^{n-2} \cos x dx.$
52.  $\int \sin^m \cos^n x dx$   
 $= \begin{cases} \frac{1}{m+n} [-\sin^{m-1} x \cos^{n+1} x + (m-1) \int \sin^{m-2} x \cos^n x dx] \\ \frac{1}{m+n} [\sin^{m+1} x \cos^{n-1} x + (n-1) \int \sin^m x \cos^{n-2} x dx], \\ (m+n \neq 0). \end{cases}$
53.  $\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx, (n \geq 2).$

54.  $\int \sin^2 x dx = -\frac{1}{2} \sin x \cos x + \frac{x}{2} + C.$

55.  $\int \cos^n x dx = \frac{1}{n} \sin x \cos^{n-1} x + \frac{n-1}{n} \int \cos^{n-2} x dx, (n \geq 2).$

56.  $\int \cos^2 x dx = \frac{1}{2} \sin x \cos x + \frac{x}{2} + C.$

57.  $\int \sin^2 x \cos^2 x dx = -\frac{1}{4} \sin x \cos^3 x + \frac{1}{8} \sin x \cos x + \frac{x}{8} + C.$

58.  $\int \tan x dx = \ln |\sec x| + C.$

59.  $\int \tan^2 x dx = \tan x - x + C.$

60.  $\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx, (n \geq 2).$

61.  $\int \cot x dx = \ln |\sin x| + C.$

62.  $\int \cot^2 x dx = -\cot x - x + C.$

63.  $\int \cot^n x dx = -\frac{1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x dx, (n \geq 2).$

64.  $\int \sec x dx = \ln |\sec x + \tan x| + C.$

65.  $\int \sec^n x dx = \frac{1}{n-1} \left\{ \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x dx \right\},$   
 $(n \geq 2).$

66.  $\int \csc x dx = \ln |\csc x - \cot x| + C.$

67.  $\int \csc^n x dx = \frac{1}{n-1} \left\{ -\csc^{n-2} x \cot x + (n-2) \int \csc^{n-2} x dx \right\},$   
 $(n \leq 2).$

68.  $\int x e^{ax} dx = \frac{1}{a^2} (ax - 1) e^{ax} + C.$

69.  $\int x^n e^{ax} dx = \frac{x^n}{a} e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx.$

70.  $\int e^{ax} \sin bx dx = \frac{1}{a^2 + b^2} (a \sin bx - b \cos bx) e^{ax} + C.$

71.  $\int e^{ax} \cos bx dx = \frac{1}{a^2 + b^2} (a \cos bx + b \sin bx) e^{ax} + C.$

72.  $\int \ln |x| dx = x \ln |x| - x + C.$

73.  $\int x^m \ln^n |x| dx = \frac{1}{m+1} \left\{ x^{m+1} \ln^n |x| - n \int x^m \ln^{n-1} |x| dx \right\},$   
 $(m \neq -1).$

74.  $\int \ln^n |x| dx = x \ln^n |x| - n \int \ln^{n-1} |x| dx.$

75.  $\int x^n \ln |x| dx = \frac{x^{n+1}}{n+1} \left( \ln |x| - \frac{1}{n+1} \right) + C, (n \neq -1).$

76.  $\int \frac{\ln^n |x|}{x} dx = \frac{1}{n+1} \ln^{n+1} |x| + C, (n \neq -1).$

77.  $\int \frac{1}{x \ln |x|} dx = \ln |\ln |x|| + C.$

78.  $\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1-x^2} + C.$

79.  $\int x^n \sin^{-1} x dx = \frac{1}{n+1} \left\{ x^{n+1} \sin^{-1} x - \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx \right\}, (n \neq -1).$

80.  $\int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln (x^2 + 1) + C.$

81.  $\int x^n \tan^{-1} x dx = \frac{1}{n+1} \left\{ x^{n+1} \tan^{-1} x - \int \frac{x^{n+1}}{x^2 + 1} dx \right\}, (n \neq -1).$

82.  $\int \sec^{-1} x dx = x \sec^{-1} x - \ln |x + \sqrt{x^2 - 1}| + C.$