LANE COMMUNITY COLLEGE MATHEMATICS DEPARTMENT

GEOMETRY

MATH REVIEW SHEETS

The following pages list key geometry concepts. Many concepts are needed to be successful in Trigonometry, Calculus, Math for Elementary Teachers, and to a lesser extent, College Algebra. The concepts summarized are taught in Geometry and in the coordinate graphing sections of pre-college algebra courses.

WHY THESE SHEETS ARE USEFUL -

* To refresh your memory on old math skills you may have forgotten.
* To review for math placement tests.
* To help you decide which math course is best for you.

HOW TO USE THESE SHEETS -

* We recommend that you cover the solution to the examples and try working the problems yourself. Check your work by looking at the solution steps.

KEEP IN MIND -

* These pages are not intended to be a short course. You should use them to simply help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place.

Alice Kaseberg
August, 1989

LANE COMMUNITY COLLEGE MATHEMATICS DEPARTMENT

MATH REVIEW SHEETS - - GEOMETRY REVIEW

Caution: As you look over the following material, keep in mind that we are summarizing material for review for the student who has had the coursework previously. Also keep in mind that geometry tends to use considerably more vocabulary than algebra courses. Thus we use extensive vocabulary in this review and make no attempt to define the words.

The goal of your study of geometry at the college level is to be able to solve problems by recognizing the geometry in a situation, drawing a reasonable picture, and applying geometric and algebraic principles.

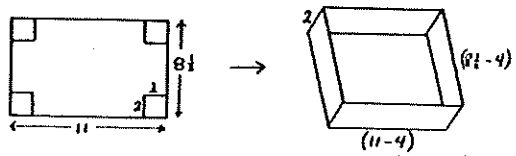
METRIC GEOMETRY (Geometry associated with measurement.)

I. Formulas for Perimeter, Area, Volume (capacity), and Surface Area. Be familiar with common units of measurement in both Standard English and Metric Systems.

Description	Formula	Variables
Perimeter (P)		
circumference (C) of a circle	$C = \pi d$	$\pi \approx 3.14$, d = diameter
rectangle	$P = 2l + 2s$	l = length, w = width
square	$P = 4s$	s = side
triangle	$P = a + b + c$	a, b, c = sides
Area (A)		
circle	$A = \pi r^2$	$\pi \approx 3.14$, r = radius
parallelogram	$A = bh$	b = base, h = height
rectangle	$A = lw$	l = length, w = width
square	$A = s^2$	s = side
triangle	$A = (1/2)bh$	b = base, h = height
trapezoid	$A = (1/2)(a + b)h$	a, b = parallel sides, h = height
Volume (V)		
cube	$V = s^3$	s = side
cone	$V = (1/3)\pi r^2 h$	r = radius, h = height
cylinder	$V = \pi r^2 h$	r = radius, h = height
pyramid	$V = (1/3)Bh$	B = area of base, h = height
rectangular box	$V = lwh$	l = length, w = width, and h = height
sphere	$V = \frac{4\pi r^3}{3}$	r = radius
Surface Area (S)		
cylinder	$S = 2\pi r^2 + 2\pi rh$	r = radius, h = height
sphere	$S = 4\pi r^2$	r = radius

Problems:

1. Find the volume of a box without lid made from an 8 1/2 x 11 inch paper with 2 inch square corners cut out.



$V = l \cdot w \cdot h$

$V = (11-4)(8\frac{1}{2}-4)(2)$

$V = 7(4\frac{1}{2})(2)$

$V = 63$ CUBIC INCHES

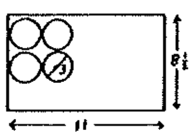
2. Express the area, A, of a circle as a function of its circumference, C.

$A = \pi r^2$

$C = 2\pi r \longrightarrow r = \frac{C}{2\pi}$ SO $A = \pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2 = \pi \frac{C^2}{4\pi^2} = \frac{C^2}{4\pi}$

THUS $A = \frac{C^2}{4\pi}$

3. An 8 1/2 x 11 sheet of paper has 4 circles of diameter 3 inches cut from it. What is the area of the remaining paper?



$d = 3 \rightarrow r = \frac{3}{2}$

AREA PAPER − AREA CIRCLES = REMAINING AREA

$l \cdot w - 4 \cdot \pi r^2 = A$

$(11)(8\frac{1}{2}) - 4 \cdot \pi \left(\frac{3}{2}\right)^2 = A$

$93\frac{1}{2} - 9\pi = A$

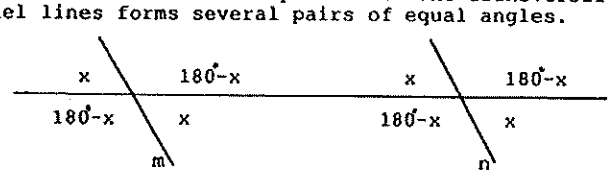
IF $\pi \approx 3.14$, $A \approx 65.24$ SQUARE INCHES

II. <u>Angle Measurement</u>

A. The angle between the rays forming an angle is measured in degrees. 360 degrees is the traditional value for a full revolution between the rays of an angle. (Another form of angle measure is radians which is introduced in Trigonometry.)

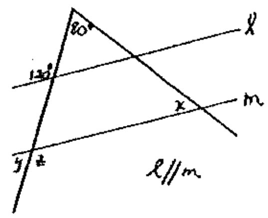
B. Supplementary angles are two angles which add to 180 degrees; complementary angles add to 90 degrees. A linear pair or two angles along a straight line are supplementary. Angles in a triangle add to 180 degrees. Angles in a quadrilateral add to 360 degrees. Angles in other polygons can be derived by placing diagonals to cut the figure into a minimum number of triangles.

C. Lines m and n below are parallel. The transversal cutting the parallel lines forms several pairs of equal angles.

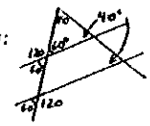


Problems:

What is the measure of angles labeled x, y, z ?



SOLUTION:



$x = 40°$
$y = 60°$
$z = 120°$

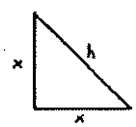
III. Pythagorean Theorem

A. A triangle is a right triangle if and only if the sum of the squares of the legs is the square of the hypotenuse. (If and only if means the statement is reversible.)

$$a^2 + b^2 = c^2$$

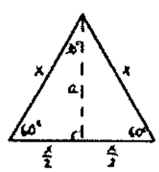
Problems: (These problems are themselves very important concepts but are included as problems because they are typical of the applications of geometry encountered in trigonometry and calculus.)

1. Show that an isosceles right triangle has sides in the ratio 1:1: $\sqrt{2}$.



$x^2 + x^2 = h^2$
$2x^2 = h^2$
$x\sqrt{2} = h$
IF SIDE $x = 1$, THEN $h = \sqrt{2}$

2. Show that a 30-60-90 degree right triangle has sides in the ratio 1: $\sqrt{3}$: 2.



DRAW ALTITUDE, a, IN EQUILATERAL TRIANGLE, FORMING TWO CONGRUENT TRIANGLES (HL THEOREM)
LET THE SIDE OF THE EQUILATERAL TRIANGLE BE X
THEN $\left(\frac{x}{2}\right)^2 + a^2 = x^2$ BY PYTHAGOREAN THEOREM.

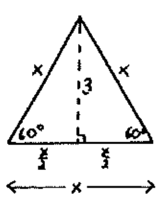
$a^2 = x^2 - \frac{x^2}{4}$

$a^2 = \frac{3}{4}x^2$

$a = \frac{x\sqrt{3}}{2}$

IF $x = 2$, $a = \sqrt{3}$, $\frac{x}{2} = 1$.

3. Find the area of an equilateral triangle with height = 3 in.



FROM #2 ABOVE

$\left(\frac{x}{2}\right)^2 + 3^2 = x^2$

$9 = \frac{3}{4}x^2$

$\frac{36}{3} = x^2$

$x = \sqrt{12}$

$A = \frac{1}{2}bh$

$b = x = \sqrt{12}$ $h = 3$

$A = \frac{1}{2}\sqrt{12}\,(3)$

$= \frac{3}{2}\sqrt{4}\sqrt{3}$

$= 3\sqrt{3}$

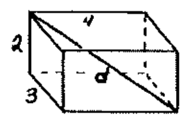
$A = 3\sqrt{3}$ SQUARE INCHES

B. The Pythagorean Theorem extends to three dimensions: In a box, the long diagonal squared is equal to the sum of the squares of the length, width, and height.

$$\ell^2 + w^2 + h^2 = d^2$$

Problems:

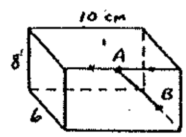
1. What is the length of the diagonal of a box with dimensions 2 x 3 x 4 inches?



$d^2 = \ell^2 + w^2 + h^2 = 4^2 + 3^2 + 2^2 = 29$

$d = \sqrt{29} \approx 5.385$ INCHES

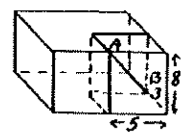
2. Find AB is A and B are midpoints of the edges of the rectangular parallelepiped (box) shown.



$\overline{AB}$ FORMS THE DIAGONAL OF A SMALLER BOX WITH DIMENSIONS $5 \times 3 \times 8$ cm.

$AB^2 = 5^2 + 3^2 + 8^2 = 98$

$AB = \sqrt{98} \approx 9.899$ cm



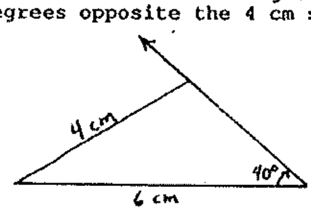
IV. Triangle Congruence

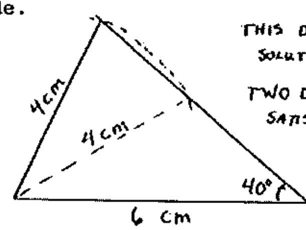
Two triangles are congruent if their corresponding sides and angles match according to Side-Angle-Side (SAS), Side-Side-Side (SSS), Angle-Side-Angle (ASA), or if in a right triangle, Hypotenuse-Leg (HL).

Triangles are not necessarily congruent if their corresponding sides and angles match according to Side-Side-Angle (SSA).

Problem:

1. Sketch a triangle with sides 4 cm, 6 cm, and an angle of 40 degrees opposite the 4 cm side.





THIS DATA FITS S.S.A. SO THE SOLUTION MAY NOT BE UNIQUE.

TWO DIFFERENT TRIANGLES SATISFY DATA.

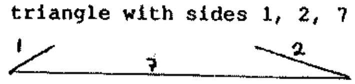
V. Triangle Inequality

The two shortest sides of a triangle must add up to more than the longest side.

Problems:

1. Sketch a triangle with sides 1, 2, 7 inches.

(TO SCALE (CM:LINE))



IMPOSSIBLE (TRIANGLE INEQUALITY)

VII. Triangle Similarity

A. Triangles with congruent corresponding angles, Angle-Angle-Angle, are called similar triangles. Corresponding sides of similar triangles are proportional.

B. Relating triangles or angles with parallel lines: Parallel lines cut off proportional line segments on transversals.

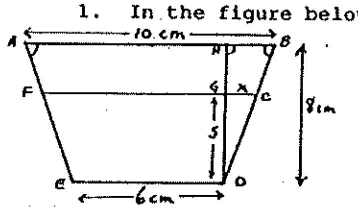
C. The Trigonometric definitions of the sine, cosine and tangent ratios are based on the proportional side property of similar triangles. Angle A is one of the acute angles of a right triangle.

sine ratio of A = $\frac{\text{opposite leg}}{\text{hypotenuse}}$ cosine ratio of A = $\frac{\text{adjacent leg}}{\text{hypotenuse}}$

tangent ratio of A = $\frac{\text{opposite leg}}{\text{adjacent leg}}$

Problems:

1. In the figure below, what is the length of segment x?



ABDE IS ISOSCELES TRAPEZOID

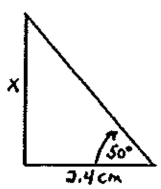
AB//CF//ED

SOLUTION: HB = 2 FROM ISOSCELES AND SYMMETRY

$\frac{5}{8} = \frac{x}{HB}$ BY SIMILAR TRIANGLES

$x = \frac{(5)(2)}{8} = 1.25$ cm

2. Using an appropriate trig ratio, find the length of side x:



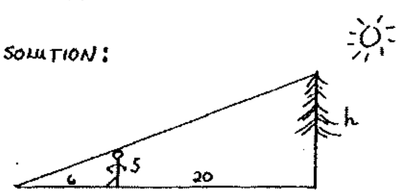
SOLUTION: $\tan 50° = \frac{x}{2.4}$

$x = (2.4) \tan 50°$

$x = 2.86$ cm

3. At a certain time on a clear day, the tip of the shadow of a tree and the shadow of a 5 foot tall girl coincide. The girl stands 20 feet from the tree. If the girl's shadow is 6 feet long, how tall is the tree?

SOLUTION:



$\frac{5}{6} = \frac{h}{20+6}$ FROM SIMILAR TRIANGLES

$6h = 5(26)$

$h = 21\frac{2}{3}$ FEET

VIII. Similarity: Area and Volume

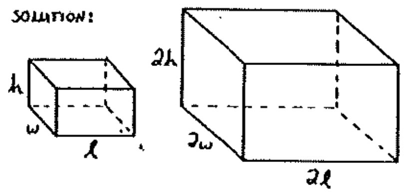
A. The ratio of the areas of two similar figures is equal to the square of the ratios of their corresponding parts (sides of linear figures, radii of circles).

B. The ratio of the volumes of two similar figures is equal to the cube of the ratios of their corresponding parts (sides of objects made with straight lines and flat surfaces, radii of spheres).

Problem:

1. If the length, width, and height of a box are doubled, how does the volume of the newly formed box compare with the original box?

SOLUTION:



$V_1 = \ell \cdot w \cdot h$ $\quad V_2 = (2\ell)(2w)(2h)$

$V_2 = 8\,\ell \cdot w \cdot h$

$V_2 = 8\,V_1$

SECOND BOX IS 8 TIMES LARGER THAN ORIGINAL BOX.

IX. Circles

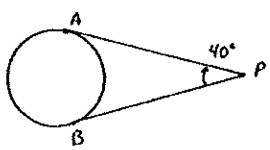
A. A tangent to a circle is perpendicular to the radius at the point of tangency.

B. Two tangents to a circle from the same exterior point are equal in length.

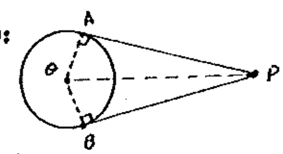
C. The central angles in degrees = intercepted arc length in degrees. The inscribed angle in degrees = 1/2 intercepted arc length in degrees.

Problem:

Lines PA and PB are tangent to a circle at points A and B, respectively. Angle APB measures 40 degrees. What is the length of the minor arc AB ?



SOLUTION:



$\angle APB = 40°$ (GIVEN) $\quad \angle PAO = \angle PBO = 90°$ (FACT A)

$\triangle APO \cong \triangle BPO$ ($\overline{AP} = \overline{BP}$, FACT B; $\overline{OA} = \overline{OB}$, RADII; $\overline{OP} = \overline{OP}$, S.S.S.)

$\angle APO = 20°$

$\angle AOP = 70°$

$\angle AOB = 140°$ = THE CENTRAL ANGLE

ARC AB $= 140°$ (FACT C)

NON-METRIC GEOMETRY (Geometry involved with vocabulary and classification)

I. Identify and classify shapes into appropriate categories.

A. Triangles: scalene, obtuse, acute, isosceles, equilateral, right.

Be able to identify and draw all medians, altitudes, angle bisectors, and perpendicular bisectors of sides for each type of triangle. Observe properties of common intersections.

Use terms such as base, height, auxiliary line.

B. Quadrilaterals: square, rectangle, parallelogram, rhombus (diamond), trapezoid, non-convex quadrilaterial, kite.

Be able to identify type of quadrilateral by relative length of sides, number of parallel sides, equality of sides, number of right angles, bisection of diagonals, or perpendicularity of diagonals.

C. Polygons: triangle, quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides, also septagon), octagon (8 sides), nonagon (9 sides), decagon (10 sides), dodecagon (12 sides).

D. Regular Polyhedra: tetrahedron (4 equilateral triangles as faces), cube (6 squares as faces), octahedron (8 eq. triangles), dodecahedron (12 pentagonal), icosahedron (20 eq. triangles).

E. Solids: pyramid, parallelepiped, rectilinear solid, cone, cylinder, sphere, prism.

F. Curved shapes: circle, ellipse (oval), parabola, hyperbola.
[These shapes are known as conic sections and are presented in detail in Intermediate Algebra and in Trigonometry. They are not in the LCC Geometry course but are described elsewhere in this packet.]

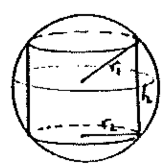
Problems:

1. Sketch a cylinder inscribed in a sphere.

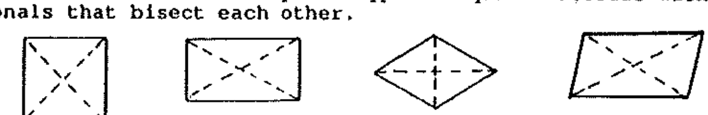
r_1 = RADIUS OF SPHERE

h = HEIGHT OF CYLINDER

r_2 = RADIUS OF CYLINDER BASE



2. Sketch and identify all types of quadrilaterals with diagonals that bisect each other.



SQUARE RECTANGLE RHOMBUS PARALLELOGRAM

3. Ignoring air or wind resistance, what is the path of a ball thrown into the air?

A PARABOLA

II. Use common terminology associated with 2 and 3 dimensional geometric figures.

A. <u>Points</u>: intersection, vertex, collinear.

B. <u>Angles</u>: acute, right, obtuse, straight, reflex, inscribed, central, interior, exterior, complementary, supplementary, vertical, adjacent, dihedral.

C. <u>Lines:</u> lines, line segment (edge), rays (half-line with one endpoint), perpendicular, parallel, skew, auxiliary lines (in triangles), coincident, concurrent.

D. <u>Parallel lines:</u> interior angles, alternate angles, corresponding angles, exterior angles.

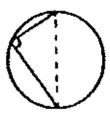
E. <u>Circles:</u> semicircle, arc, sector, segment, chord, diameter, radius, radii, center, tangent, secant, inscribed angle, central angle, inscribed circles, circumscribed circles.

F. <u>Planes:</u> parallel planes, intersecting planes, lines of intersection, point of intersection, half plane.

G. <u>Shapes:</u> congruent, similar, oblique.

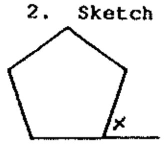
Problems:

1. What kind of angle is formed if the angle is inscribed in a circle and the sides of the angle pass through the endpoints of the diameter of the circle?



A RIGHT ANGLE
(THE INTERCEPTED ARC IS 180° SO THE ANGLE MEASURES 90°)

2. Sketch an exterior angle of a regular pentagon.



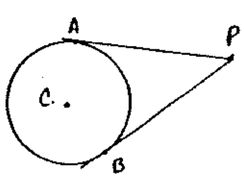
X IS THE EXTERIOR ANGLE

THE MEASURE OF X CAN BE FOUND BY FINDING THE SIZE OF EACH INSIDE ANGLE AND SUBTRACTING FROM 180°.

$x = 180° - \frac{3(180)°}{5}$

$x = 72°$

3. Sketch a circle with center, C, and two tangents to the circle from an exterior point, P. [From the metric geometry section, what do you know about the tangent line and the radius at point of tangency? What do you know about the distance from P to the two tangent points?]



SOLUTION

$CA \perp PA$, $CB \perp PB$

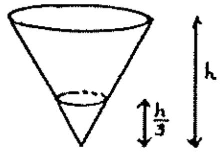
$PA = PB$

III. <u>Visual perception including cross sections</u> of solids based on preceding vocabulary.

Problems:

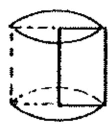
1. Sketch a cone-shaped tank resting on its point with water level at 1/3 of its height.

SOLUTION:



2. Sketch a rectangle and the shape obtained by rotating it about one of its long edges.

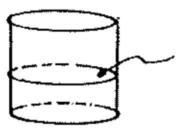
SOLUTION:



OBTAIN A CYLINDER

3. If the answer in problem 2 is sliced parallel to the short side of the rectangle, what shape is obtained?

SOLUTION:



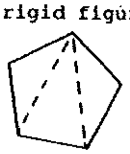
CROSS SECTION IS A CIRCLE

IV. <u>Explore rigidity of a figure</u>.

Problems:

1. What is the minimum number of diagonals needed in a pentagon to make it a rigid figure?

SOLUTION:



TWO DIAGONALS WILL "TRIANGULATE" THE PENTAGON AND MAKE IT RIGID.

2. What is structurally wrong with this gate?

SOLUTION: THE GATE NEEDS A DIAGONAL TO MAKE IT RIGID.

V. Symmetry

College Algebra and Trigonometry emphasize the symmetry properties of equations and functions both in algebraic terms and in graphing.

A. Line symmetry, reflection, or axial symmetry refer to the property of an object being able to be reflected over or folded along a line and match with itself. The axis of symmetry or line of symmetry is frequently the x or y axis in coordinate graphing.

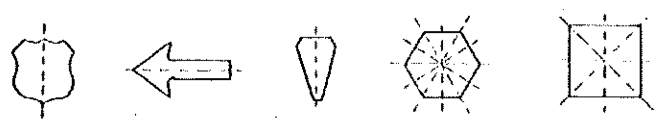
B. Rotation, or rotational symmetry, refers to the property of an object being able to be rotated about a point and remain unchanged. The center of symmetry is frequently the origin in coordinate graphing. (The rectangular hyperbola and simple third degree equations have point symmetry with the point of symmetry being the origin.)

C. Translation or slide symmetry refers to the motion where one shape moves to another location without rotating or changing in any way.

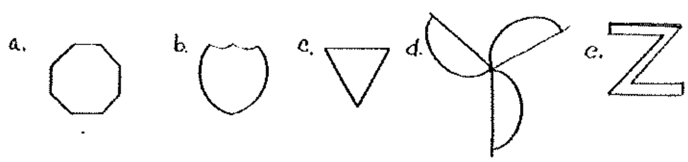
Problems:

1. Draw in the line or axis of symmetry in each figure. There may be more than one in each figure.

SOLUTION:



2. Which of these have rotational symmetry?



SOLUTION: a, c, d, e

COORDINATE GRAPHING or ANALYTIC GEOMETRY

[This topic is commonly part of both algebra and geometry curricula in secondary schools. At LCC, it is presented in Introductory Algebra and Intermediate Algebra, expanded in College Algebra and used heavily in Trigonometry and Calculus. In the latter two courses as well as in engineering courses, students are expected to be able to assign coordinates to geometric figures and derive formulas or other information.]

I. Vocabulary:
Coordinate (x,y), x-axis (horizontal axis), y-axis (vertical axis), origin, quadrant, x-intercept (a,0), y-intercept (0,b).

II. Slope:

A. Slope of lines = $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

B. Special slopes:

Horizontal lines have zero slope.
Vertical lines have undefined slope due to division by zero.
Parallel lines have the same slope.
Perpendicular lines have slopes that are negative reciprocals; hence, the slopes of perpendicular lines multiply to a negative one.

III. Linear Equations

A. Slope-intercept form: $y = mx + b$ where m = slope, b = y-intercept

B. Point-slope form: $y - y_1 = m(x - x_1)$

where m = slope, and (x_1, y_1) = any point on the line.

C. Two point form: use two points to find the slope then use point-slope form above.

IV. Distance Formula: (Pythagorean Theorem applied to coordinates)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

V. Midpoint Formula: (The average of two coordinates)

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Problems (Sections I to V, above)

1. Find the equation of the line through (4,5) and (2,-3).

SLOPE $= \frac{5-^-3}{4-2} = \frac{8}{2} = 4$ USE EITHER POINT IN $y-y_1 = m(x-x_1)$

$(4,5) \rightarrow y-5 = 4(x-4)$ $(2,-3) \rightarrow y+3 = 4(x-2)$

$y = 4x-16+5$ $y = 4x-8-3$

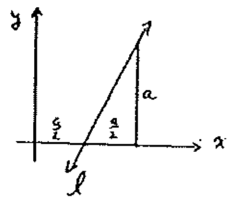
$y = 4x-11$ ← SAME → $y = 4x-11$

2. Are the lines y = 3x and 3y + x = 0 perpendicular?

YES THEIR SLOPES, 3 AND $-\frac{1}{3}$, ARE NEGATIVE RECIPROCALS

($y = 3x$ $3y + x = 0$ is $3y = -x$ or $y = -\frac{1}{3}x$)

3. For the figure, find an equation for the line, ℓ, in the form y = mx + b.



SOLUTIONS SLOPE $= \frac{a}{\frac{a}{2}} = 2$, POINT (a,a) and $(\frac{a}{2},0)$

$y - y_1 = m(x-x_1)$

$y - 0 = 2(x - \frac{a}{2})$

$y = 2x - a$