FUNDAMENTAL

$$\sin^2 x + \cos^2 x = 1$$

$$\left\{\begin{array}{l} \sin^2 x = 1 - \cos^2 x \\ \sin x = \pm\sqrt{1 - \cos^2 x} \\ \cos^2 x = 1 - \sin^2 x \\ \cos x = \pm\sqrt{1 - \sin^2 x} \end{array}\right\}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}, \quad \sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left\{\begin{array}{l} 1 + \cot^2 x = \csc^2 x \\ \tan^2 x + 1 = \sec^2 x \end{array}\right.$$

ADDITION

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

DOUBLE ARC

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

HALF ARC

$$\left\{\begin{array}{l} \sin^2 x = \dfrac{1 - \cos 2x}{2} \\ \sin x = \pm\sqrt{\dfrac{1 - \cos 2x}{2}} \end{array}\right.$$

$$\left\{\begin{array}{l} \cos^2 x = \dfrac{1 + \cos 2x}{2} \\ \mathrm{cox}\ x = \pm\sqrt{\dfrac{1 + \cos 2x}{2}} \end{array}\right.$$

$$\tan x = \frac{\sin 2x}{1 + \cos 2x}$$

$$\tan x = \frac{1 - \cos 2x}{\sin 2x}$$

PRODUCT

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)$$

$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)$$

SUM

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

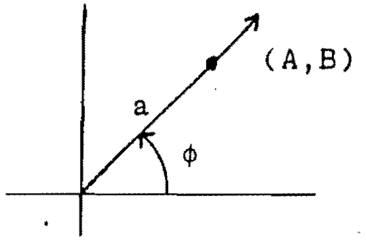
$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

REDUCTION OF $A\cos\theta + B\sin\theta$

For the diagram below, $a = \sqrt{A^2 + B}$



$$A = a \cos \phi$$

$$B = a \sin \phi$$

For all values of θ,

$$A\cos\theta + B\sin\theta = a\cos(\theta - \phi)$$