

**Hyperbolic Functions:
Derivatives and Integrals**

$$\frac{d}{dx}[\sinh u] = (\cosh u) \frac{du}{dx}$$

$$\int (\cosh u)u' dx = \sinh u + C$$

$$\frac{d}{dx}[\cosh u] = (\sinh u) \frac{du}{dx}$$

$$\int (\sinh u)u' dx = \cosh u + C$$

$$\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u) \frac{du}{dx}$$

$$\int (\operatorname{sech}^2 u)u' dx = \tanh u + C$$

$$\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u) \frac{du}{dx}$$

$$\int (\operatorname{csch}^2 u)u' dx = -\coth u + C$$

$$\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u) \frac{du}{dx}$$

$$\int (\operatorname{sech} u \tanh u)u' dx = -\operatorname{sech} u + C$$

$$\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u) \frac{du}{dx}$$

$$\int (\operatorname{csch} u \coth u)u' dx = -\operatorname{csch} u + C$$

Inverse Hyperbolic Functions

Domain

$$\text{Inverse hyperbolic sine: } \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \quad (-\infty, \infty)$$

$$\text{Inverse hyperbolic cosine: } \cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad [1, \infty)$$

$$\text{Inverse hyperbolic tangent: } \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad (-1, 1)$$

$$\text{Inverse hyperbolic cotangent: } \coth^{-1} x = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right) \quad (-\infty, -1) \text{ and } (1, \infty)$$

$$\text{Inverse hyperbolic secant: } \operatorname{sech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1-x^2}{x^2}}\right) \quad (0, 1]$$

$$\text{Inverse hyperbolic cosecant: } \operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1+x^2}{x^2}}\right) \quad (-\infty, 0) \text{ and } (0, \infty)$$

**Inverse Hyperbolic
Functions:
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$$\left. \begin{aligned} \frac{d}{dx}[\sinh^{-1} u] &= \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx} \\ \frac{d}{dx}[\cosh^{-1} u] &= \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx} \end{aligned} \right\} \int \frac{u'}{\sqrt{u^2 \pm a^2}} dx = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$\left. \begin{aligned} \frac{d}{dx}[\tanh^{-1} u] &= \frac{1}{1-u^2} \frac{du}{dx} \\ \frac{d}{dx}[\coth^{-1} u] &= \frac{1}{1-u^2} \frac{du}{dx} \end{aligned} \right\} \int \frac{u'}{a^2 - u^2} dx = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$\left. \begin{aligned} \frac{d}{dx}[\operatorname{sech}^{-1} u] &= \frac{-1}{u \sqrt{1-u^2}} \frac{du}{dx} \\ \frac{d}{dx}[\operatorname{csch}^{-1} u] &= \frac{-1}{|u| \sqrt{1+u^2}} \frac{du}{dx} \end{aligned} \right\} \int \frac{u'}{u \sqrt{a^2 \pm u^2}} dx = -\frac{1}{a} \ln \left(\frac{a \pm \sqrt{a^2 \pm u^2}}{|u|} \right) + C$$

