ANSWERS TO PRACTICE OF ALGEBRA FOR CALCULUS.

Some of these problems are parts of problems that you will have to solve this term. All these problems should be solved without a calculator.

A Factor out the following expressions. Write your answers without rational or negative exponent.

Example: An answer such as $(x-1)^{-\frac{1}{2}}(x-2(x-1))$ should be rewritten as $\dfrac{2-x}{\sqrt{x-1}}$

$$x+x^2+x^3=x\left(1+x+x^2\right)$$

$$x^3+x^4+x^5=x^3\left(1+x+x^2\right)$$

$$x^{-3}+x^{-2}+x^{-1}=x^{-3}\left(1+x+x^2\right)=\frac{1+x+x^2}{x^3}$$

$$x^{\frac{1}{2}}+x^{\frac{3}{2}}+x^{\frac{5}{2}}=\sqrt{x}\left(1+x+x^2\right)$$

$$x^{-\frac{1}{2}}+x^{\frac{1}{2}}+x^{\frac{3}{2}}=\frac{1+x+x^2}{\sqrt{x}}$$

$$x^{-\frac{3}{2}}+2x^{\frac{1}{2}}+x^{\frac{5}{2}}=\frac{\left(x^2+1\right)^2}{\sqrt{x^3}}$$

$$(x+1)^{\frac{1}{2}}+(x+1)^{\frac{3}{2}}=\sqrt{x+1}(x+2)$$

$$(x+1)^{-\frac{1}{2}}+(x+1)^{\frac{1}{2}}+(x+1)^{\frac{3}{2}}=\frac{x^2+3x+3}{\sqrt{x+1}}$$

$$x(x+1)^{-\frac{1}{2}}+x(x+1)^{\frac{1}{2}}+x(x+1)^{\frac{3}{2}}=\frac{x\left(x^2+3x+3\right)}{\sqrt{x+1}}$$

$$x^2(x+1)^{-\frac{1}{2}}+2x(x+1)^{\frac{1}{2}}+(x+1)^{\frac{3}{2}}=\frac{(2x+1)^2}{\sqrt{x+1}}$$

$$\left(2-x^3\right)^{-\frac{1}{2}}\left(-3x^2\right)\left(3-x^3\right)^2-\left(2-x^3\right)^{\frac{1}{2}}2\left(3-x^3\right)\left(-3x^2\right)=\frac{-3x^2(x-1)(x^2+x+1)(3-x^3)}{\sqrt{2-x^3}}$$

$$\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)\left(2x^2+1\right)+\left(1-x^2\right)^{\frac{1}{2}}(4x)=\frac{x\left(3-6x^2\right)}{\sqrt{1-x^2}}$$

$$(2x-3)^2+(2x-3)^3+4x^2(3-2x)=2(2x-3)(3-5x)$$

B Simplify

$$R(x)=\frac{(1-x^3)(x^2-2x+4)(4x^2+14x+12)(x^2+x^3)(4x^2+4x-3)}{(x^3+8)(8x^2+24x+18)(x-x^3)(x^2+x+1)(2x^2-x)}=1$$

$$Q(x)=\frac{(12xy^3-3x^3y)(2x^3y-x^2y^3)(4x^2+2xy^2+y^4)}{(8x^3-y^6)(x^3y+2x^2y^2)(6xy^2-3x^2y)}=1$$

$$2^{100}-2^{99}=2^{99}$$

$$\sqrt[18]{\frac{3^{(3^3)}}{(3^3)^3}}=3$$

$$\frac{1}{\sqrt[m]{\frac{2^{n-m}}{2^{m+n}}}}-2=2$$

$$(2x-3y)(2y-3x)-(3y-2x)(3x-2y)=0$$

C Factor as much as possible

$$x^2-1=(x-1)(x+1)$$
$$x^3-1=(x-1)(x^2+x+1)$$
$$x^4-1=(x-1)(x+1)(x^2+1)$$
$$x^{16}-1=(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)$$
$$8x^6y^9-27z^3=(2x^2y^3-3z)(4x^4y^6+6x^2y^3z+9z^2)$$
$$6x^2-18x^4+3x=3x(2x-6x^3+1)$$

$$\frac{3}{x^3}+\frac{y}{x}-\frac{2}{x^4}=\frac{1}{x^4}(3x+x^3y-2)$$

$$\frac{a}{b^2}-\frac{3ac}{b^3}+\frac{b}{a^3}-\frac{c}{a^2b^4}=\frac{1}{a^3b^4}(a^4b^2-3a^4bc+b^5-ac)$$

Special factorization

$$x^3+2x+y=y\left(\frac{x^3}{y}+\frac{2x}{y}+1\right)$$
$$x^3+2x+y=x^2\left(x+\frac{2}{x}+\frac{y}{x^2}\right)$$
$$x^3+2x+y=x^3\left(1+\frac{2}{x^2}+\frac{y}{x^3}\right)$$
$$x^3+2x+y=y\left(\frac{x^3}{y}+\frac{2x}{y}+1\right)$$

D Simplify the following fractions. No fraction in numerator and denominator.

Example: $\dfrac{\frac{1}{2}x+2x^2}{x-5}=\dfrac{\frac{x+4x^2}{2}}{x-5}=\dfrac{x(4x+1)}{2(x-5)}$

$$\frac{\frac{x}{2}(x-1)-2(x-1)}{x}=\frac{(x-1)(x-4)}{2x}$$

$$\frac{x}{2}(x-1)^{-\frac{1}{2}}-2(x-1)^{\frac{1}{2}}=\frac{4-3x}{2\sqrt{x-1}}$$

$$\left(2-x^3\right)^{-\frac{1}{2}}\left(-x^2\right)\left(3-x^3\right)^2-\left(2-x^3\right)^{\frac{1}{2}}\left(3-x^3\right)\left(\frac{1}{2}x^2\right)=\frac{x^2\left(3-x^3\right)\left(3x^3-8\right)}{2\sqrt{2-x^3}}$$

$$\frac{-p\left(\frac{2}{3}\right)\left(\frac{20}{p}\right)^{-\frac{1}{3}}\left(-\frac{20}{p^2}\right)}{\left(\frac{20}{p}\right)^{\frac{2}{3}}}=\frac{2}{3}$$

E Simplify $E(x)$ using the DIFFERENCE OF TWO CUBES identity and THEN evaluate $E\left(\frac{1}{\sqrt{3}}\right)$.

$E(x)=(x+1)^3-(x-1)^3$ **Check your answer by directly evaluating $E\left(\frac{1}{\sqrt{3}}\right)$.**

F Review the properties of logarithms summarized below and simplify the following expressions.

$\log_b xy=\log_b x+\log_b y$ $\log_b \frac{x}{y}=\log_b x-\log_b y$

$\log_b x^n=n\log_b x$ $\log_e x=\ln x$

$\log_b 1=0$ $\log_b b=1$ $\ln e=1$

$\log_b \frac{1}{x}=\log_b 1-\log_b x=-\log x$

$\log_b\left(b^x\right)=b^{(\log_b x)}=x$ if $x>0$ (Why this restriction?)

$\log 10 =$ 1

$\log 1000 =$ 3

$\log 0 =$ *DNE*

$\log .001 =$ -3

$\log_7 7 = 1$

$\log_7 49 = 2$

$\log_7(-7)$ *DNE*

$\log_2 2 = 1$ $\qquad \log_2 \frac{1}{2} = -1$ $\qquad \log_2 8 = 3$

$\log_2 \frac{1}{8} = -3$ $\qquad \log_2 \sqrt{2} = 1/2$ $\qquad \log_2 \frac{1}{\sqrt{2}} = -1/2$

$\log_2 \sqrt{8} = 3/2$ $\qquad \log_2 \frac{1}{\sqrt{8}} = -3/2$ $\qquad \log_2 = \left(-\frac{1}{\sqrt{8}}\right) = \text{DNE}$

$\log_3 \sqrt{\frac{1}{27}} = -3/2$ $\qquad \log_2 \frac{1}{\sqrt[5]{16}} = -4/5$ $\qquad \log_3 \sqrt{\sqrt{\sqrt{3}}} = 1/8$

$\ln e = 1$ $\qquad \ln e^2 = 2$ $\qquad (\ln e)^2 = 1$

$\ln\left(\frac{1}{e^2}\right) = -2$ $\qquad \left(\ln \frac{1}{e}\right)^2 = 1$ $\qquad \ln\left(\frac{1}{e}\right)^2 = -2$

$\left(\ln\left(\frac{1}{e^2}\right)^2\right)^2 = 16$ $\qquad \ln(1-e) = \text{DNE}$ $\qquad \ln(e^e) = e$

$\log_3 3^7 - 2^{\log_2 7} = 0$ $\qquad \log_b c - \frac{1}{\log_c b} = 0$ $\qquad e^{(\ln x + 1)} - ex = 0$

$\ln\left(\frac{\ln x + \ln x^2 + \ln x^3 + \ln x^4}{\ln x} - 9\right) = 0$ $\qquad e^{\ln(x+1)} + e^{(\ln x + 1)} - 1$

$\ln(x^2 - x + 1) - \ln(x^3 + 1) + \ln(x+1) = 0$ $\qquad \frac{x \ln x - \frac{\ln x}{x}}{x} = \ln x \frac{x^2 - 1}{x^2}$

$\frac{x}{x \ln x - \frac{x}{\ln x}} - \frac{1}{(\ln x)^2 - 1} = \frac{1}{\ln x + 1}$ $\qquad \frac{\ln \frac{1}{e} - \left(\ln \frac{1}{e^2}\right)^2}{(\ln e)^2 - \ln e^2} = 5$

G **If** $\log_b 2 = a$, $\log_b 3 = c$, and $\log_b 5 = d$ **in some base b, express the following in terms of** a, b, and c.

$\log_b 30 = a + c + d$ $\qquad \log_b 100 = 2(a + d)$ $\qquad \log_b 300 = 2a + c + 2d$ $\qquad \log_b \frac{45}{2} = 2c + d - a$

$\log_b \sqrt{6} = \frac{a+c}{2}$ $\log_b \frac{3\sqrt{2}}{5} = \frac{a}{2} + c - d$ $\log_b \frac{5\sqrt{3}}{2} = \frac{c}{2} + d - a$ $\log_b \frac{\sqrt[5]{5}}{5} = \frac{-4d}{5}$

H **Solve for x.**

$\ln(\ln(\ln(\ln(\ln x)))) = 0$ $x = e^{e^{e^e}}$

$e^{\ln x + 1} - ex = 0$ $S = (0, \infty)$

$e^{2x} - 2e^x + 1 = 0$ $x = 0$

$e^x - 2 + e^{-x} = 0$ $x = 0$

$\log_2 x = m$ $x = 2^m$

$\log_x 9 = \frac{2}{3}$ $x = 27$

$\ln(x-1) + \ln(x+2) = \ln(x+1)$ $x = \sqrt{3}$ only

$\log_{1789} x - \log_{1789}(x-1) = \log_{1789}(x+1) - \log_{1789} 2$ $x = 1 + \sqrt{2}$ only

I **If an investment doubles in 10 years, how long does it take to triple? You may assume that the interest is either compounded continuously or not. How long does it take to quadruple?**

$\frac{10\ln 3}{\ln 2}$ years. 20 years.

Show that **any integer** N can be written using exactly three times the digit 4 and any mathematical tools available in math 111 by showing that

$$N = -\log_4\left(\log_4 \sqrt{\sqrt{\sqrt{\cdots\cdots\cdots\sqrt{\sqrt{4}}}}}\right)$$ with 2N radical signs.

$$-\log_4\left(\log_4 \sqrt{\sqrt{\sqrt{\cdots\cdots\cdots\sqrt{\sqrt{4}}}}}\right) = -\log_4\left(\log_4 4^{\left(\left(\frac{1}{2}\right)^{2N}\right)}\right) = -\log_4\left(\log_4 4^{\left(\frac{1}{4^N}\right)}\right) =$$

$$-\log_4 \frac{1}{4^N} = N$$