

simulation"; that it was "an informative class that will leave you feeling like you have learned ample amounts but will make you want more"; and that they "learned as much from the other students as I did from the instructor."

Simulations that interact with and change according to contemporary events are a different twist on an already widely used technique, but this approach really seems to work. It is time intensive, assumes a certain degree of risk because one never knows for sure what will happen in the real world, and requires a great deal of flexibility, but it engages students with their course materials and the world around them in ways that can be extraordinary.

To Discuss or Not to Discuss: Integrating Pedagogies for Honors and Mathematics

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INTRODUCTION

Classroom discussion has long been an important pedagogical tool in fields such as the social sciences and the humanities because it allows students to critically examine ideas about what it means to be a human living within communities of other humans. It allows students to formulate ideas and judgments and to reconsider these formulations as time passes and new information is revealed. Within the field of mathematics, however, professors typically rely on lectures to ensure coverage. While students are asked to think critically, they are likely asked to do so within the context of homework problems. The primary question we consider in this essay is whether class discussion enriches student thinking and problem-solving processes within the field of mathematics.

The University Honors Program at Southern Polytechnic State University has found itself engaged in answering questions concerning the benefit of discussion as a tool for critical thinking, creative thinking, and problem solving since its inception in 2003. At that time an honors committee, composed of seven people representing the four schools, honed the policies and procedures for the University Honors Program. This committee set curriculum guidelines designed to ensure that courses involve substantially more tangible content than typical courses at the same level in at least four of the following seven areas: creative thinking, critical thinking, problem solving, oral or written communication, collaborative work, experiential learning, and interdisciplinary components.

For the humanists and social scientists on the committee, the idea of discussion-based classes in which students actively engage in the exchange of ideas generated through creative thought, critical thought, and problem solving seemed a promising solution for meeting many of these guidelines. The scientists and mathematicians instead saw the use of mathematical problems and assigned laboratory experiments as an optimal way to engage students in creative thought, critical thought, and problem solving. These differences in pedagogy spurred quite a bit of discussion among the honors committee members about the different methods the various fields use to emphasize student engagement in learning.

No standard for the use of discussion was created. However, many of the reasons articulated for using discussion-based pedagogy are supported by research on discussion-based classes. Stephen D. Brookfield and Stephen Preskill list many benefits in *Discussion as a Way of Teaching*, such as:

1. Discussion helps students develop critical thinking skills as they scrutinize their own assumptions as well as the assumptions of others.
2. It helps to create intellectual agility since students must think on their feet and react to unanticipated comments.
3. Discussion helps students develop skills of synthesis and integration because they must listen and speak as they work to understand how information is related. (24–35).

Discussion-based pedagogy functions in a similar manner to the Saxon Math Pedagogy, which draws on research showing that continual practice and review help students to demonstrate more thorough understanding of the material being taught (Mayfield and Chase). Just as, in class discussion, students are continually engaged with ideas that are reviewed and refined as the semester progresses.

Not until the honors office received the evaluations for L. R. Ritter's fall of 2007 Honors Calculus One class did members of the honors community re-evaluate discussion-based pedagogy. Evaluations for Ritter's class revealed not only that students in the Honors Calculus One class thought Ritter was one of the best mathematics instructors with whom they had ever worked but also that they felt the discussions were stimulating and contributed to the quality of the class.

The director decided to create a more formal study concerning the use of discussion as a tool for learning. In August 2008, the honors program hosted workshops and roundtables on the use of discussion, and these activities led William Griffiths also to implement discussion within the Honors Calculus One class. Ritter and Griffiths, along with Nancy Reichert, Director of the University Honors Program, have been involved in both initiating and documenting this process of implementation.

At the heart of our study is exploration of the following questions: What does it mean to run a "discussion-based" mathematics class? How is this discussion balanced with other techniques such as lecture or individual problem-solving? What pre-existing, cultural understandings about a field's knowledge base and its pedagogy lead to the ways in which both teacher and students approach learning? Our answers are drawn from the multiple perspectives of the teachers, students, and honors director.

THE DIRECTOR'S PERSPECTIVE

Because my background entails literary criticism and rhetorical theory, I look at discussion-based pedagogy through the concepts of language constructs

and knowledge-making. Ferdinand de Saussure's linguistic theory explores language as a system of signs whose values are determined by relationships set up by the users of these signs in order to facilitate communication. What Saussure emphasizes is that signs are arbitrary; they have no direct relationship with their referent. Therefore, a gap always exists between the signifier (whether audible or written) and the signified (that to which the signifier points). Thus, there is no absolute quality of "treeness" for the word "tree"; only because a community of people has come together and agreed that the word "tree" represents a certain type of vegetation do we understand "tree" to signify the thing it represents. That other languages (systems of signs) such as French, German, and Spanish have other words to signify this particular type of vegetation helps us to understand that the relationship between the signifier and the signified is arbitrary.

In English, the arbitrary nature of the signifier is especially clear when words like "bad" can in one instance refer to something being "morally evil" or "unacceptable" and in another instance refer to something being "extremely good." Even when signifiers seem more tied to the signified, they can still symbolize something else entirely. For example, "tree" can symbolize "life" or a family genealogy.

Like languages, knowledge-making is both an individual and communal act; knowledge is not constant but is ever changing as individuals within the community and the community itself bring new knowledge to light.

A belief that language has arbitrary qualities and that knowledge is not already set but is instead always being made as we speak has an important impact on how we see classroom dynamics. While students are receivers of knowledge that is communicated by the professor (such as standard language conventions for a field of study), they are also makers of knowledge. For example, when I was teaching in Texas in the spring of 1997, my students coined the term "Bubbonics" for the ways they used language. They felt that their being "bubbas" influenced their language choices and their ability to use Standard English much in the same way that African Americans have been influenced by Ebonics. Around the same time that my students were using the term, it was also being used by Jim Wright in the *Dallas Morning News* for similar reasons (McFedries). Language is in constant flux as is knowledge-making itself. Discussion-based pedagogy allows for exploration of knowledge and language. Therefore, discussion becomes an important part of thinking and learning as we investigate the nature of language even as we use it to formulate ideas and communicate them to each other.

Such concepts are important to a paper on discussion-based pedagogy and mathematics because math can also be seen as a system of signs with a process of signification using arbitrary signifiers that refer to the signified. However, math attempts to erase the gap between the signifier and the signified. Although mathematics includes some variations in convention, particularly with respect to notation, many mathematical terms are not allowed any gradation of meaning. For example, an algebraic structure known as a *group* must always satisfy

a fixed set of axioms without respect to any specific application, context, or cultural consideration. Mathematicians typically try to curtail any fluidity of language; they use their “language” to solve problems and explore mathematical knowledge. However, they can still use discussion to allow students to explore problem-solving within a group of like-minded individuals and to formulate ideas, test them, and reformulate them as the need arises.

L. R. RITTER'S PERSPECTIVE, CALCULUS ONE AND TWO, 2007–08

At the opening of the fall 2008 semester, I was caught off guard by the director of the honors program here at Southern Polytechnic State University when she asked me how I had managed to facilitate a discussion-based class during the previous fall and spring semesters in the Honors Calculus sequence. I agreed to participate in a roundtable discussion on discussion-based teaching—a methodology preferred in honors humanities and social science courses. As I had not set out to conduct a discussion-based class, I had to look back and analyze the classes that I had taught. I also sought the input of some of the students who had participated in the classes, for it was the students in those courses who brought the discussion-based approach in my classes to the attention of the honors director.

I had taught the honors version of the Calculus sequence during the 2007–08 academic year. Enrollment was restricted to students in the honors program or to students who had special permission. The class sizes were very small—seven in the fall and twelve in the spring—as compared to a typical course with thirty-six students at Southern Polytechnic. I had not entered into teaching the sequence with an intention to conduct discussion-based classes. My intention in differentiating these classes from the non-honors version was to incorporate a higher degree of mathematical rigor and a greater degree of emphasis on written and oral communication skills. I also hoped to defuse the view of mathematics as a one-course-at-a-time sequence of readily solvable problems. I wanted students to explore mathematics in a broader human and historical perspective.

In a consideration of discussion-based teaching in mathematics, some important issues come to mind. For me, the first is how to ensure that some of the basic material, such as definitions, theorems, and techniques that aren’t “up for discussion,” are conveyed to and mastered by the students. This issue is of special interest in courses like Calculus One and Two, which are primarily prerequisites for further study. As a practicing mathematician, I understand the value of brainstorming, discussion, and debate about problem-solving techniques and the validity of assumptions and arguments, so another question is how to facilitate discussion when most students don’t expect it and might not (at least initially) be comfortable with it in a classroom.

In every class I teach, I try to maintain a relaxed and open atmosphere that invites questions and participation, so I was interested in identifying any aspects

of this course sequence in particular that heightened the open atmosphere and created a sense of being *discussion-based*. I reviewed the teaching techniques I used over the year and in addition asked three students who had participated in those classes to serve as case studies and to give their perspectives.

I identified a number of aspects of the class including teaching and assessment tools that I believe promoted a discussion atmosphere. The small class sizes and take-home quizzes both helped to promote a discussion atmosphere. Each student received a particularly challenging problem at the end of the week with the expectation that he or she would present a solution (or partial solution) to the class the following week. After the first of these sessions, the class seemed to develop an increased sense of cohesiveness. All students had to undergo the same stress and were supportive of one another. When a student stopped short of a complete solution, the class members were free to offer input. A similar teaching tool that I used was collaborative in-class quizzes. Students were given the option to participate or to take a traditional solo quiz. During the spring semester, among twelve students only two regularly opted out of group activity. Students engaging in the group work, which would take place over a fifty-minute class period while I sat off in the corner, were able to take advantage of each other’s strengths. Finally, students were strongly encouraged to ask questions, make observations, and answer questions posed by each other as well as myself.

To get insight into the students’ experience, I received input from three of the participating students, who, although all traditional students, represented a broad spectrum of personality types. Chris was home schooled, extremely bright and creative, but also shy and reserved. Michael came from a public high school. He was bright, imaginative, and extremely gregarious. Marcus was highly intelligent with strong leadership skills. I asked them all a few questions and encouraged them to provide any additional input.

I began with the following question: In your opinion (i.e. without regard to some formal definition of *discussion-based classes* or *lecture-based*), would you say that your Calculus One and Two classes were (a) pure lecture, (b) mostly lecture, (c) half-n-half, (d) mostly discussion, or (e) pure discussion. Of the three students, one responded “half-n-half,” and two responded “mostly lecture,” so it would be unusual to characterize the sequence as *discussion-based*. However, I also asked the students to identify any class characteristic or activities that gave the classes a discussion orientation. All three identified class size as a primary contributor. Chris, who is somewhat shy, wrote:

I think it’s all about smaller class sizes. It’s a lot easier to ask questions when you only have to worry about wasting the time of seven other people, rather than thirty. It’s also a lot easier to risk saying something dumb (like asking a stupid question or replying to question) when there are fewer people to embarrass yourself in front of.

Other aspects of the classes identified by the students as contributing to a discussion orientation were: (1) questions posed by the instructor, (2) questions posed by other students, (3) collaborative quizzes, (4) and a cooperative classroom environment. They also agreed that the collection of personalities was a key component to the open atmosphere of the classes.

When asked to compare the classes with other mathematics courses they had taken, all three students reported spending more or much more time answering questions, writing on the chalkboard, and talking about topics tangential to the course. Two reported talking more or much more about topics not related to the course. This last revelation does not surprise me; however, it does raise again the question of whether such a highly active and participatory classroom setting can distract students from the material, and so I asked, "If the class(es) seemed discussion-oriented to you, do you think this enhanced your understanding of the material, distracted you from the material at hand, or had little impact on what you got out of the course?" All three students reported that it enhanced their understanding. Marcus in particular stated: "While I don't believe the discussion helped me with my grades, it did help me to truly understand what it is we were doing and why we were doing it." Chris made a similar statement about the role of discussion: "Since most of the discussion was geared towards related material, I think it helped me develop a deeper, more well-rounded understanding of the course material."

An important objective of mine was to give the students a glimpse into mathematics beyond the simple scope of the course without the emphasis being placed on the grade. All three students reported learning more or much more in the honors sequence about mathematics in general ("not confined solely to course topics") than in other mathematics courses. Michael, who is inquisitive, wrote: "I loved those Honors Math classes. I ALWAYS looked forward to class, never skipped (unless I had to study for a class I was shake-y about), and wanted to stay later and learn more." Although some people might suggest that attempts to implement discussion and other nontraditional methodologies in the mathematics classroom are often met with resistance from the students, it was the students' positive feedback that motivated my further use of discussion techniques in the following semesters. Michael's statement reflects not only an acceptance of such methods but an enthusiasm for them.

WILLIAM GRIFFITHS'S VIEW (CALCULUS ONE) FALL 2008

As the fall semester approached, I found myself both concerned and enthused about the Honors Calculus One teaching opportunity. I wondered how exactly I would answer this challenge. When I agreed to teach the course, I knew there would be a discussion component but not exactly what that meant. My "traditional" courses already incorporated discussion. I had read material and heard certain buzzwords: be a "guide-on-the-side" instead of a "sage-on-the-stage," "shut up and teach," and so forth. Had I applied this to my

regular classes to a sufficient extent, and how could I distinguish an honors section?

Questions were followed by new ideas. Perhaps I should attempt to have the students lead the discussion. I could try sitting among the students, perhaps turning the chairs toward one another. We could discuss ideas, theories, and proofs. With a small class, we might be able to get away from the blackboard. Honors students, perhaps, would require less repetitious homework. In order to facilitate the discussion, I could have the students read the text for a particular lesson before class since reading a mathematics text has a value all its own. Almost no students actually read their math text because they really learn by solving problems, so this would be different. I also could have them attempt to solve problems before discussing the material to encourage more concrete discussion.

I did attend a roundtable discussion hosted by the honors program on the subject of discussion-based honors courses before the semester began. However, at this session, those representing other departments stated several times that my course, given the subject matter, would probably have less discussion than theirs; they were unsure that their techniques could be applied in a mathematics course.

At this point the real battle started. I needed to make a syllabus. The one idea the honors faculty all agreed on is that a discussion-based course should have clear guidelines for the assessment of participation. Being a numbers person, I liked the idea of a point system, but I did not want competition over points, at least not in a serious fashion, since it would defeat the goal of collaboration. I liked the idea of rewarding all students when the class led the discussion, but I didn't know if that would actually happen. It was difficult to plan when I could only guess at the dynamic, but I resolved to distribute points over the semester. Also, in order to avoid unfriendly competition, I decided to let the students know that I intended to ensure that all of the students would receive most of their participation grade.

I had already developed a very nice homework list for use in my calculus courses. I provided the honors students with the same list but did not require them to solve all of the problems. Instead, I proposed that the students go through the list and choose problems, giving them some control and also avoiding needless repetition; I figured that these students likely did not need to solve six problems of the same type when two would do. For assignments, I decided to collect a subset from the homework list, supplemented by more challenging problems. Beyond that, I decided to have the same number of exams as a regular calculus class, on approximately the same dates, with one or two extra sections of material thrown into each unit. With a plan firmly in hand, I began the semester.

As I presented my ideas on the first day, the students seemed largely pleased with the way I intended to run the course. Students who had never taken calculus before voiced some minor concerns. They felt the discussion-based course

might be better suited for those with more experience with calculus. I should also mention here the make-up of the class: of the twelve students, ten were first-semester freshmen, and only two had previously taken an honors course at SPSU. Those few who had previously been in a discussion-based university course were unsure that the same procedures could be applied to their math class but were willing to try.

In the first week, it became apparent that I had made some mistakes. Allowing free choice of problems seemed like a great idea, but I overlooked the possibility that few students would attempt the same problems. Participation and effort were good, but we did not have enough for discussion as a class. I quickly decided to choose a few of the later homework problems for each section to be attempted by all. I confess to being not a little frightened at this point. In trying to do things differently, I had overlooked a consequence that should have been rather obvious. What else was in store?

By the second week, the class was not progressing very well. Each day, I entered the class, took a desk, and turned it toward the students. I asked them for any questions or comments they had about their reading. Silence. Did they have any difficulties with any of the problems? Silence. I tried waiting them out, and eventually someone would say something, but mostly to break the silence. I could sense they were waiting for me. I could get things started, but they were so used to a right answer, one way to do things, that discussion would end if we solved a problem. It is difficult to go against their "programming" of a dozen years of math courses with right answers and the teacher's method of getting there. I needed to show them that there are more ways than one to solve a problem, and limits (the current topic of discussion) were not the best topic for this purpose; a limit asks for the value a function approaches as its input approaches a number rather than the actual value obtained by the function at that input number. Solving these problems is fairly intuitive, and solutions to early problems on the topic are strikingly similar.

For me, class was dragging a bit at this point, and if I was uncomfortable, it was likely worse for the students. So, I decided to bend a little. I decided to stand at the front of the classroom, fine. I decided to interject a bit more on proper terms and usage. I returned to leading the discussion myself. But I decided not to lecture. As in all things, I discussed this matter with the students since I like to be as transparent as possible in what I am trying to do. And oh, yes, I let them know that they would be frequently coming to the board!

As the semester progressed, this format seemed to function adequately. It was not what I had originally thought of as a full-discussion course, but the students got more comfortable and talked more. As more time passed, it bothered them less to have me sit among them while a classmate presented a proof of a theorem. Soon it was acceptable for me to sit near the board and talk with them about problems and applications, while making notes when necessary, instead of writing everything for them to copy down. By request, I would do a problem for which they had no more ideas, but I let them know that I preferred not to

do so. I preferred to help the students get an idea that they could take home with them to puzzle out. I believe that since they had an option, it was easier for them to agree to discuss and try again. I started to think that beginning the semester full-swing in a discussion-based style had been another obvious miss of mine; maybe they needed to be eased into it. Certainly there was an adjustment period in which they became more comfortable asking questions of one another, and this also took time.

As the semester came to an end, we did have a good group dynamic. I let them create their own assignment, and they decided to construct optimized cereal bowls in various shapes. A student suggested minimizing the required surface area for various shapes with a given volume and then constructing these optimized "bowls" for display. The fact that students met with me four days a week and also that most of the students lived in the same dorm encouraged collaboration. They developed a friendly competition for points; instead of being pitted against one another, the competition became more like a race. They knew they did not need to refute one another for points, and I gave them points for assists, and so the competition became a game of one-upmanship, leapfrog if you will. Admittedly, we had lost four students along the way, but competence was high among the remaining students. Most importantly, the stronger students were helping and encouraging those of lesser ability, and exam scores improved as well.

Finally, parts of the course had worked, and others had not. At the end of the semester, I asked the students to fill out surveys for me, and I also discussed the course with them and how it was taught. The most interesting statistic, I think, is that all of the passing students chose to continue with me into Honors Calculus Two the following semester, so we spent an hour talking about what they did and didn't like about the course and where we could make improvements.

I am happy to say that the students had several excellent suggestions. They really enjoyed the point distribution that I used for participation; it was concrete and fun. However, they said they wanted to see a little more structure to it. My method had been simple; I threw out points at random intervals and offered points for proving theorems. The students expressed a preference for a weekly quota to ensure that they kept up with it throughout the semester rather than thinking they might make up more points later when they were better prepared. We all ended up feeling that this change would improve the overall preparedness for discussion.

One major change the students suggested was somewhat surprising to me. They thought that reading the book was helpful and worthwhile, but one of the difficulties we had faced early on with the simpler problems arose because I had them reading and working problems, together and individually, before class; consequently, most of the discussion points had been cleared up before class began. The students felt that it would be better to read over the material and examples, discuss them in class, and then solve the problems. I believe this suggestion will make the discussions livelier and will help the students solve

the more difficult problems in a timelier manner and with fewer interjections from me.

Another major concern was the level of difficulty. I admit that, when I took on an honors course, my immediate compulsion was to increase the difficulty rather significantly. The students preferred to see an honors course as different rather than more difficult. Since I always seek to challenge my students, I challenged these students on several levels and perhaps overwhelmed them at times. This is an issue I will have to think about for the future. While I do believe that an honors course will, and should, be more difficult than a regular course, I wonder how much of the difficulty should arise from the material and how much should result from the clearer and deeper knowledge of the student.

To get more insight into how students reacted to the discussion-based Calculus One class, I asked them to fill out a survey about the class discussion. Table 1 indicates how students responded. One point was given for each answer in which students "strongly disagreed," two points were given for each answer in which students "disagreed," three points for each "neither agreed nor disagreed," four points for each "agreed," and five points for each "strongly agreed." An average for each item was then created.

Overall, the students agreed that the discussion improved the learning experience. When we consider this information within the context of the student answers for item #5, we can see that students did come into the course with ideas about the usefulness of discussion for mathematics classes, yet they still were able to find the discussion useful. Answers to item #7 seem to further indicate that the students and I were coming to terms with what it means to run a successful discussion-based math class. The results of the survey indicate that the class opened their minds to possibilities for how discussion could be used to help them succeed. The fact that all of the students will be continuing on with me into Calculus Two leads me to conclude the students did succeed and learn in this atmosphere.

The SIRS, a standard evaluation instrument used at SPSU, as well as the honors student course evaluations solidified my conclusion that the class had succeeded. On the SIRS evaluation, this course was rated over the mean for comparative four-year institutions in every category. A "5" is the highest rating one can receive. In the overall evaluation category, the course was rated a 4.29, over the comparative mean of 3.99. The most statistically significant result among the categories was in student effort and involvement. In this section, the course was rated 4.19 over the comparative mean of 3.70, indicating that the course is adequate in substance to challenge honors students without adding an excess of extra material.

For the honors student course evaluation, students rate the course on the same scale we used in the survey above. On these evaluations, the students indicated overwhelmingly that they were challenged by the course as well as having studied and prepared. For the statement "My learning increased because of this course," the students rated an average of 4.375, and if we remove the

Table 1: Honors Calculus I Survey, Fall 2008

Statement	Strongly Disagree	Disagree	Neither Agree nor Disagree	Agree	Strongly Agree	Average
1. I believe that the discussions from this course enhanced the learning experience.			1	4	2	4.14
2. I would have preferred a more traditional, lecture-based course.	1	1	2	2	1	3.14
3. The discussion element of the course was too forced, i.e. it could have evolved more naturally.		1	5	1		3.00
4. I was more focused in this course by the discussion element.		4	1	1	1	2.86
5. Math courses are an appropriate choice for a discussion-based course.		2	3	1	1	3.14
6. I learned more from this class through the interactions with other students than I would have in a more traditional course.	1	2		4		3.00
7. Some improvements in the design of this course would more easily facilitate my being involved in the discussion.			2	5		3.71
8. I do not like discussion-based courses.	1	3	2	1		2.43

one student rating of 3 or below on preparation, the average becomes 4.71. Another positive result was a 4.5 average on agreement with the statement "The course helped me to think independently about the subject matter." Among the free response comments from the students, the most common (at least six of the eight students) were that the class was kept interesting through discussion, the class was challenging, and the expertise of the professor was praiseworthy. Only one student raised concerns about the discussion-based pedagogy.

My experiment with discussion-based math classes has been intriguing. I have been at times frustrated, angry, hopeful, and elated. I have seen both strengths and weaknesses in what we have done. I shall certainly be attempting to refine the process for the next semester and to continue testing this course on different classes in the future. As for our class, and the sequel to follow, the students and I find great value as we come to understand what it means to use discussion well in a mathematics course.

CONCLUSIONS DRAWN FROM OUR STUDY

Our multiple perspectives and experiences have shown us that discussion can play an active role in mathematics classes, especially in helping students understand more deeply the mathematical concepts and theory that pertain to the field. That said, all three of us learned that we needed to better define the term "discussion-based," including how much of class time must be spent in discussion for a class to be "discussion-based."

Like William Taylor in his article "Promoting Critical Thinking through Classroom Discussion," those of us using discussion in the honors program at SPSU are interested in helping students to think through their ideas and to problem solve. Like Taylor we believe that discussion helps students to "practice forming their own judgments, and to do so in an atmosphere that is safe, supportive, and instructive" (78). Taylor articulates a four-step process:

1. Students formulate an initial judgment: "One that results from a careful analysis of text and data."
2. Students then articulate their "initial, tentative judgment" along with the "data and perspectives" that inform it to the rest of the class.
3. Students receive new "data and perspectives" from the discussion. They can then modify or reaffirm their original judgment.
4. Finally the students articulate their modified views. Either consensus will be reached or the participants will become clear on why they cannot agree. (78-79).

In the SPSU honors math classes, we saw students engaged in this process: forming judgments, not just individually but in small groups, sharing perspectives and data in order to solve calculus problems, and presenting their ideas to the rest of the class.

We know that Ritter's and Griffiths's discussion techniques fit the standards set by Taylor. However, both teachers raised concerns about how to use discussion successfully in the mathematics classroom; Ritter focused on the need for lecture in order to explain specific mathematical concepts, and Griffiths focused on easing students into the discussion-based format. Taylor can help us here once again. He has created the following continuum to show who controls the conversation in the classroom and who determines the "right" answers (79):

Teacher Control		Student Control	
Monologue	Quiz Show	Discussion	Bull Session

Teachers dominate classes when they lecture, thus creating a monologue. The "Quiz Show" allows a student to enter the conversation, but only by answering narrow questions with right answers. "Discussions" are focused conversations since students are guided by the faculty to make informed judgments. Finally "Bull Sessions" are student-controlled discussions that may be arbitrary and random.

Ritter's and Griffiths's concerns seem to be about balancing the teacher/student control of the classroom. Ritter is concerned with how to use lectures for definitions, theorems, and techniques and how to move beyond simple "quiz shows" for student discussion. Griffiths's concern, not seen on Taylor's continuum, is the silence that occurs when students wait for the teacher to tell them what to think.

Both Griffiths and Ritter have created similar "alternative images" to that which Taylor articulates in his article. Taylor finds that an alternative image to the one in which the teacher sits in a student desk and contributes minimally to the conversation is one in which the teacher is at the "center of the discussion, orchestrating its dynamics by calling on students and pointing out the connections between what is being said" (81). Griffiths's evolving discussion-based instruction in the end created a continuum that began with a student-led framework, evolved into an alternative image, and then evolved back to a framework that was once again more student-led.

Ritter's alternative image combines various levels along the continuum for a discussion-based class; lectures or even mini-lectures using the dynamic of questions and connections helped students to begin thinking on the issues and concepts for the day while small-group work and full-class discussions immersed students more fully in the course material, allowing students to problem-solve and formulate judgments.

Clearly discussion-based mathematic classes can offer students many of the same opportunities found in other fields. However, they can do so only to the extent that professors are willing to engage their students in such an exploration within the classroom. What may stand in the way are the pre-existing notions of both students and faculty about how mathematical material should

be presented. In our case, both students and faculty had previously been taught by other mathematicians who used a lecture-based format. Given that Ritter's excellent evaluations led to the formation of this study and that Griffiths' evaluations for the Honors Calculus One class were also excellent, it is clear that pre-existing notions of how mathematics should be taught can be successfully challenged. In the spring of 2009, both faculty members were nominated by students from their honors classes for the Honors Faculty Member of the Year Award. These nominations may indicate that students are even more willing to work with a new format than previously thought.

The title of our article begins with a false dilemma: "to discuss or not to discuss." The false dilemma fallacy considers only two alternatives when in fact there are others. In teaching of honors mathematics, more options exist than discussion or lecture; the options are part of a continuum such as that employed by Taylor.

Despite the success of our experiment with discussion-based math classes, several questions remain unanswered. For example, Ritter asks how we should introduce discussion-like pedagogy into courses dominated by lecture. On several levels we did not answer this question. First of all, even though the honors program held workshops and a roundtable discussion in the fall, faculty still felt at odds about what a "discussion-based" class would look like for a mathematics class. Definitions, models, and techniques were grounded in ideas gleaned from Stephen D. Brookfield and Stephen Preskill's book *Discussion as a Way of Teaching* as well as ideas from Donald Finkel's book *Teaching with Your Mouth Shut*. Honors faculty also modeled and spoke on different methods and techniques they used to stimulate discussion. However, the term "discussion-based" remained troubling for several reasons. How much discussion makes a class "discussion-based"? What are all of the definitions, models, and techniques that are essential to the term "discussion-based"?

We realized that we were talking about the term "discussion" as though we all shared a consistent definition but that it was a rather nebulous term. It was like the proverbial elephant, with each of us grabbing onto a different part and thinking we understood the whole. Some of us have been using the same basic discussion techniques for so long we see them as "natural" and have a hard time stepping back from them in order to name them for others or to try new techniques. Others of us are so new to the idea of discussion and so trapped in other techniques that seem "natural" that we cannot begin to envision the use of discussion techniques for our classes.

The next step for our program is to go back to definitions, models, and techniques in order to articulate more fully what our terms mean and how we use discussion to facilitate learning in our fields of study. We will also track honors mathematics students when possible to see how their success in discussion-based classes affects in any way their success in future classes.

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