Sabbatical Report of Phil Moore

My sabbatical during winter term of 2012 was granted for the purpose of working on a book in progress on the topic of the history and theory of *perfect numbers*. Perfect numbers are those whole numbers which are equal to the sum of their proper factors, so for example, 6, equal to 1 + 2 + 3, and 28, equal to 1 + 2 + 4 + 7 + 14, are the first two perfect numbers. A topic of mathematical interest at least since the time of Euclid around 300 BCE, they have been the subject of scrutiny, speculation, and study from antiquity through the Middle Ages up through the birth of modern mathematics in the 17th century and even today in the age of computers. Thousands of enthusiastic amateurs have contributed to the search for new perfect numbers, and I conceive of this book as a means for people such as these amateur contributors, as well as potential contributors and interested non-contributors, to learn about the fascinating mathematics and history of this subject.

My previous work in this area consisted of a draft document outlining the history of the topic from the time of Euclid to modern times, plus a few incomplete sections explaining the technical mathematics in more detail, to be eventually included in the complete work as appendices for those who were interested in such details, but not essential for reading and enjoying the rest of the work. My original plan for the sabbatical was to finish these technical sections, and to do some correspondence with researchers who might be able to shed more light on some of the historical details of the past few decades, and to reformulate the work for submission to publishers. I did not intend to do much substantive new research on the topic, aside from the correspondence with researchers, but as I reviewed my draft and surveyed recent research on this subject, I discovered that a substantial amount of research has been done recently (within the past 3 years) relevant to topics covered in this book. A number of these recent research papers have addressed the possible existence or non-existence of odd perfect numbers. One recent paper proves that odd perfect numbers must have at least 9 distinct prime factors. Another proves that any odd perfect number must have at least 101 total prime factors, including repeated factors. Another key paper proves that any odd perfect number must have at least 1500 digits in our base-10 decimal system, a distinct improvement over the previously published limit of at least 300 digits. Because of this wealth of new results in this area, I spent an extensive time during my sabbatical studying these key papers and corresponding with the authors. As all of these papers rely on computational results, one of the key questions I raised with the authors was how much computational effort was required to establish these results, and how much effort would be required to improve the results.

I did spend a considerable amount of time editing and rewriting my original draft to create a more readable and engaging text. Most of my writing effort during my sabbatical centered on the period from 300 BCE to 1800 CE, corresponding to roughly the first half of the book, and I did succeed in extensively rewriting this portion of the text. I had originally conceived of this effort as purely an editing and rephrasing effort, but as I again studied the original sources and reflected on my draft, I discovered that when I had written the draft, I had overlooked an important facet of an essential mystery of the origin of this subject. Even though I had originally seen the mystery of where this topic of perfect numbers had come from as the primary mystery, Euclid clearly relates perfect numbers to the topic of prime numbers and the factorization of any whole number into its prime factors. My stunning realization in the progress of this sabbatical project was that the origin of this entire theory of prime numbers was also a complete mystery, and the origin of the theory of perfect numbers was only a small part of this greater mystery. Unfortunately, I discovered that there are very few hints in the literature predating Euclid. Plato and Aristotle appear to be the only definitive sources, along with some fragments on the history of mathematics quoted by later writers, but even among these sources, the hints are only tangential and elusive. An extensive study of Greek mathematics and its history became a major focus of this sabbatical. In addition, I discovered that there was apparently no historical record of the concept of a prime number before Euclid in the Babylonian, Egyptian, or Chinese traditions, so my conclusion is that the concept of prime number was probably original to Greek mathematics in roughly the century or so before Euclid.

48 perfect numbers are currently known. All of these perfect numbers are even and follow a formula first described by Euclid about 300 BCE. It has been conjectured that there are an infinite number of even perfect numbers. The Great Internet Mersenne Prime Search (GIMPS) is currently conducting an internet-based search for even perfect numbers, and has been successful in discovering 14 new perfect numbers since 1996. Euler proved around 1750 CE that all even perfect numbers followed Euclid's formula, but his result left open the question of whether any odd perfect numbers might exist. Although no one has proved that an odd perfect number is impossible, a number of theorems have shown that the existence of an odd perfect number is at best unlikely. The question of the existence of odd perfect numbers has been described as possibly the oldest unsolved problem in mathematics, and is a major focus of my manuscript. My hope is that this book may serve not only as an inspiration for amateurs contributing to the search for even perfect numbers, but also as an inspiration for individuals to either find an odd perfect number or to prove that such an odd perfect number does not exist.

Remaining work on this book will consist of revising the later chapters and incorporating the recent research results and correspondence with researchers from the past few decades into a rewritten and more dynamic narrative. My intended audience consists of enthusiastic amateurs, and my sincerest hope is that this work will serve an educational purpose, inspiring readers to further interest in and investigation of mathematics.